DELHI PUBLIC SCHOOL, RANCHI

Assignment (2016 -17)

Class:-XI **Subject:- Mathematics**

SET THEORY

1. Write the following set in roster form:-

   {x/x is a positive integer less than 10 and $2^x - 1$ is an odd number}

2. Find the value of n(s)+n(p) where s = {x/x is a positive multiple of 3 less than 100} and p = {x/x is a prime number less than 20}.

3. Make correct statements by filling in the symbols $\subseteq$ or $\not\subseteq$ in the blank spaces:-

   (a) {a,b,c}……………..{b,c,d}

   (b) {x/x is a circle in the plane} …………… {x/x is a circle in the same plane with radius 1}

   (c) {x/x is an equilateral triangle in a plane} ………….{ x/x is a triangle in the same plane.}

   (d) {x/x is a student of class XI of your school}………....{x/x is a student of your school.}

   (e) {x/x is a student of your school} ………….{x/x is a student of class XI of your school}.

4. Two finite sets have m and n elements. The total number of subsets of first set is 112 more than the total number of subsets of second set. Find the value of m and n.

5. Assume that P (A) = P (B). Show that A=B.

6. Two finite sets have m and n elements respectively. The total number of subset of first set is 56 more than the total number of subset of the second set. Find the values of m and n.

7. Is it true for any sets A and B, P (A) ∪ P(B) = P(AUB) ? Justify your answer.

8. Let P= $\left\{\frac{1}{x} : x \in N, x < 7\right\}$ and Q=$\left\{\frac{1}{2x} : x \in N, x \leq 4\right\}$ Find P∩Q.

9. If A= {x/x = 2n, n $\in Z$} and B= {x/x =3 n, nϵ Z } then find A ∩B.

10. Show that A∪ B = A∩ B ⇒A=B

11. Let A,B and C be the sets such that A∪ B = A ∪ C and A∩ B = A∩ C , show that B=C.

12. Show that A∩ B = A∩ C need not imply B=C.

13. For any sets A and B, show that P (A∩ B) = P (A) ∩ P(B).

14. For any two sets A and B, Prove that A ∩ (A∪ B)’ = ∅.

15. For any two sets A and B, Prove that (i) A ∪ (B-A) = AUB (ii) (A∩ B) U (A-B) = A.

16. For any two sets A and B, Prove that

    (A-B) U (B-A) = (AUB) – (A∩ B)

17. For any three sets A,B and C, prove that (i) (A-B) U (A-C) = A – (B ∩ C) (ii) (A-B) U (A-C) = A – (B ∪ C)

18. For any three sets A, B and C, prove that

    (i) A ∩ (B-C) = (A∩ B) - (A∩ C)

    (ii) A- (B-C) = (A-B) U (A∩ C)

19. **In a group, 150 students know Hindi and 60 know English and 10 know Hindi and English. If there are 30 students who know neither of two languages. How many students are the in the group?**

20. **In a group of 40 students , 26 take samosa, 18 take burger and 8 take neither of the two. How many take both samosa and burger?**

21. **A market research group conducted a survey of 1000 consumers and reported that 720 consumers liked product A and 450 consumers liked product B. What is the least number that must have liked both products?**

22. **Out of 500 car owners investigated 400 owned car A and 200 owned car B, 50 owned both A and B cars. Is this data correct?**

23. **In a town of 10,000 families, it was found that 40 % families buy newspaper X, 20% families buy newspaper Y and 10 % families buy newspaper Z, 5% families buy X and Y, 3 % buy y and Z and 4% buy X and Z . If 2% families buy all the three papers, find the number of families which buy**

(i) X only (ii) Y only (iii) None of X, Y and Z.

24. **In an examination, question number 1 was attempted by 67 students, question number 2 by 46 students and question no.3 by 40 students. 28 students attempted both question no. 1 and 2 ; 8 attempted both question number 2 and 3; 26 attempted both questions 1 and 3; 2 students attempted all the three questions. Find how many attempted question no. 1 but not 2 and 3.**

25. **There are 2000 students in a school. Out of these, 1000 play cricket, 600 play basketball and 550 play football. 120 play cricket and basketball, 80 play basketball and football, 150 play cricket and football , 45 play all the three games. How many students play none of the games?**

26. **In a group of 84 persons, each plays at least one game out of three viz tennis, badminton and cricket. 28 of them play cricket, 40 play tennis and 48 play badminton. If 6 play both cricket and badminton and 4 play tennis and badminton and no one plays all the three games, find the number of persons who play cricket but not tennis.?**

27. **A survey of 500 television viewers produced the given information; 285 watch football, 195 watch hockey, 115 watch cricket , 45 watch football and cricket, 70 watch football and hockey, 50 watch cricket and hockey, 50 do not watch any of the three games. How many watch exactly one of the three games?**

28. **From 50 students taking examinations in Maths, Physics and chemistry, each of the students has passed in at least one of the subject, 37 passed Maths, 24 passed physics and 43 chemistry. At most 19 passed Maths and Physics, at most 29 Maths and chemistry and at most 20 physics and chemistry. What is the largest possible number that could have passed all three examinations?**

Relation and Functions

1. State whether each of the following statements is true or false. If the statement is false, rewrite the given statement correctly.

   (i) If P={m,n} and Q= {n,m} then PXQ = {(m,n) (n,m) }

   (ii) If A and B are non-empty sets, then AxB is a non-empty set of ordered pairs (x,y) such that x ∈B and Y∈ A.

   (iii) If A={1,2}, B={3,4} then AX (B∩ ∅) =∅

2. Let A={x,y,z} and B= {1,2} . Find the number of relations from A to B.

3. Let A= {1,2,3} . Find the number of relations from A to A.

4. Let R be a relation from Q to Q defined by R={ (a,b) : a,b ∈ Q and a-b ∈ z} show that

   (i) (a,a) ∈ R ∀ a ∈ Q

   (ii) (a,b) ∈ R ⇒ (b,a) ∈ R.

   (iii) (a,b) ∈ R and (b,c) ∈ R ⇒ (a,c) ∈ R

5. Let R be a relation from N to N defined by R={(a,b): a,b, ∈ N and a=b^2 } Are the following true?

   (i) (a,a) ∈ R ∀ a ∈ N.

   (ii) (a,b) ∈ R ⇒ (b,a) ∈ R

   (iii) (a,b) ∈ R and (b,c) ∈ R ⇒ (a, c) ∈ R

   Justify your answer in each case.

6. Write the total number of function from set A to set B, where

   (i) A = {1,2,3,} , B= {a,b,c}

   (ii) A= {1,2,3}, B= {a,b,c,d}

   (iii) A= {1,2,3,4} , B= {a,b,c}

7. Let N be the set of natural numbers and the relation R be defined on N. Such that R= {(x,y): y=2x, x,y ∈ N}. What is the domain, co-domain and range of R? Is this relation a function?

8. Redefine the function which is given by

   f(x) = $|x-1| + |1+x|$, $-2 \leq x \leq 2$.

9. Find the domain of the following functions:-

   (i) f(x) = $\frac{1}{4-x^2}$ (ii) f(x) = $\frac{x^2}{1+x^2}$

   (iii) f(x)= $\sqrt{9-x^2}$ (iv) f(x) = $-|x-2|$

   (v) f(x) = $\frac{|x-1|}{x-1}$

10. The relation f is defined by f(x) = $\begin{cases} x^2, \text{if } o \leq x, \leq 3 \\ 3x, if\ 3 \leq x \leq 10 \end{cases}$

    and the relation g is defined by g(x) = $\begin{cases} x^2, \text{if } o \leq x, \leq 2 \\ 3x, if\ 2 \leq x \leq 10 \end{cases}$

    show that f is a function and g is not a function .

TRIGONOMETRY

1. Convert the following degree measures into radian measures:-

   (a) 25° (ii) 39.375° (iii) 40°20' (iv) 5°37'30" (v) $-$ 47°30'

2. If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

3. Find the angle between hour hand and minute hand of a clock at 3:30pm.

4. Find the angle between hour hand and minute hand of a clock at quarter to five.

5. Find the value of the trigonometric functions:-

   (a) Sin(-240°) (ii) Sin($-\frac{11\pi}{3}$) (iii) Cos 1110° (iv) Cos(- 1710°) (v) Cos $\left(\frac{5\pi}{3}\right)$

6. Find the value of tan $\frac{\pi}{8}$.

7. Prove that: $8\cos^3\frac{\pi}{9} - 6\cos\frac{\pi}{9} = 1$

8. Prove that : cos18° - sin 18° = $\sqrt{2}$ sin 27°

9. Prove that : $2\cos\frac{\pi}{13} \cdot \cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$

10. Prove that sin20° . sin 40°, sin 60°. Sin80° = $\frac{3}{16}$.

11. Prove that: $\frac{\sin x - \sin y}{\cos x + \cos y} = \tan\left(\frac{x-y}{2}\right)$

12. Prove that: $\frac{\sin 7x - \sin x}{\sin 8x - \sin 2x} = \frac{\sec 5x}{\sec 4x}$

13. Prove that: $(\cos x+\cos y)^2 + (\sin x-\sin y)^2 = 4\cos^2\left(\frac{x+y}{2}\right)$

14. Prove that: $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

15. Prove that: $\tan 4x = \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

16. Prove that: $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

17. Prove that: $\frac{\sin(A-C)+2\sin A+\sin(A+C)}{\sin(B-C)+2\sin B+\sin(B+C)} = \frac{\sin A}{\sin B}$

18. Prove that: $\frac{\cos 6x+6\cos 4x+15\cos 2x+10}{\cos 5x+5\cos 3x+10\cos x} = 2\cos x$

19. Prove that: $\cos^2 x+\cos^2\left(x+\frac{\pi}{3}\right)+\cos^2\left(x-\frac{\pi}{3}\right) = \frac{3}{2}$

20. Prove that: $\frac{\sec 8x-1}{\sec 4x-1} = \frac{\tan 8x}{\tan 2x}$

21. Prove that: $\frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x} = \frac{1}{2}(\tan 27x - \tan x)$

22. Prove that: $\sqrt{2+\sqrt{2+2\cos 4x}} = 2\cos x, \text{where } 0 < x < \frac{\pi}{4}$

23. Solve:- cos3x-sin2x=0

24. Solve:- sin2x – sin 4x + sin 6x =0

25. In any $\triangle$ ABC , if the angles are in the ratio 1:2:3, prove that the corresponding sides are in the ratio 1:$\sqrt{3}$:2

26. Prove that a(sin B-sin C) + b (sin c-sin A) + c (sin A – sin B) =0

27. In any $\triangle$ ABC, prove that b cos B+ c cos C = a cos (B-C)

28. In any $\triangle$ ABC, prove that

$$\frac{b^2-c^2}{cosB+Cos\,C}+\frac{c^2-a^2}{\cos c+\cos A}+\frac{a^2-b^2}{Cos\,A+cosB}=0$$

29. In any $\triangle$ ABC, prove that

Sin $\left(\frac{B-C}{2}\right)=\left(\frac{b-c}{a}\right)cos\,\frac{A}{2}$.

30. In any $\triangle$ ABC, prove that: a(cos C-Cos B)=2(b-c) $\cos^2\frac{A}{2}$

31. In any $\triangle$ ABC, prove that: $\frac{c}{a+b}=\frac{1-tan\frac{A}{2}\,.\,tan\frac{B}{2}}{1+tan\,\frac{A}{2}.tan\,\frac{B}{2}}$

32. In any $\triangle$ ABC, prove that: $\frac{c}{a-b}=\frac{tan\frac{A}{2}+\,tan\frac{B}{2}}{tan\,\frac{A}{2}-tan\,\frac{B}{2}}$

33. In any $\triangle$ ABC , prove that: a^2 sin (B-C) = (b^2-c^2). sin A.

34. In any $\triangle$ ABC, prove that: (b-c) cot $\frac{A}{2}$+ (c-a) cot $\frac{B}{2}+(a-b)cot\,\frac{C}{2}=0$

35. If a cos A = bcos B, then prove that the triangle is either isosceles or right angled.

36. In any $\triangle$ ABC , prove that: 4 $\left(\text{bc}\cos^2\frac{A}{2}+\text{ ca}\cos^2\frac{B}{2}+ab\cos^2\frac{C}{2}\right)$= $(a+b+c)^2$

37. If in any $\triangle$ ABC, $\frac{b+c}{12}=\frac{c+a}{13}=\frac{a+b}{15}$ then prove that $\frac{cos\,A}{2}=\frac{cos\,B}{7}=\frac{cos\,C}{11}$

38. In any $\triangle$ ABC if $\angle C$ = 60° , prove that $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$

39. In any $\triangle$ ABC prove that:

(b^2-c^2) cot A + (c^2-a^2) cot B + (a^2-b^2) cot c =0

40. In any $\triangle$ ABC prove that : $\left(\frac{b^2-c^2}{a^2}\right)$ sin 2A $+\left(\frac{c^2-a^2}{b^2}\right)$ sin 2B+$\left(\frac{a^2-b^2}{c^2}\right)$ sin 2C = 0

LINEAR INEQNALITIES

1. Solve for x: $\frac{1}{2}\left(\frac{3x}{5}+4\right)\ \frac{1}{3}(x-6)$

2. Solve for x: $\frac{2x-1}{3}\ \frac{3x-2}{4}\ -\ \frac{2-x}{5}$

3. Solve for x and show the graph of solution on the number line:- $\frac{3x-4}{2}\ \frac{x+1}{4}-1$

4. Solve for x and draw the graph of solution on the number line:

   (a) 3x-7 > 2 (x-6), 6-x > 11 – 2x

   (b) 5(2x-7) −3 (2x+3)0, 2x+19≤6x+47

5. Solve for x : $|2x-5| > 1$

6. Solve for x:

   (a) 1 |x-2| 3

   (b) |x+1| +|x|>3

   (c) $\frac{|x+3|+x}{x+2} > 1, x \neq -2$

7. A person is not filling well, so he goes to a doctor. The doctor on examination finds that his temperature varies between 30° c and 35° c. What is the range of temperature in Fahrenheit if conversion formula is given by C= $\frac{5}{9}$(F-32).

8. In drilling world's deepest hole it was found that the temperature T in degree celcius, x km below the earth's surface was given by T=30 + 25 (x-3), 3 x 15. At what depth will the temperature be between 155° and 205°C.

9. The water acid in a pool is considered normal when the average P^H reading of three daily measurements is between 7.0 and 7.6. If the first two P^H readings are 7.48 and 7.36, find the range of P^H value for the third reading that will result in the acidity level being normal.

10. Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

11. A solution of 8 % boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 lites of 8 % solution, how many litres of the 2% solution will have to be added?

12. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

13. Solve the following inequalities graphically:-

    (a) x+2y 0

    (b) 3x-6 0

(c) 2x-y>1, x-2y< -1.

(d) 5x+4y 40, x 2, y 3

(e) 3x+2y $150,\ x+4y\ \ 80,\ x\ \ 15,\ \ y\ \ 0$

(f) x+2y $10,\ \ x+y\ \ 1,\ \ x-y\ \ 0$, x 0, y 0

MULTIPLICATION PRINCIPLE, PERMUTATIONS AND COMBINATIONS

1. How many numbers are there between 99 and 1000 having at least one of their digits as 7?
2. Find the value of n, if $\underline{|n+1} = 12\underline{|n-1}$
3. If $\frac{1}{\llcorner 7} + \frac{1}{\llcorner 6} = \frac{1}{\llcorner 8}$, $find\ x.$
4. Find n if $n-1_{p_3} : n_{p_4}$ = 1:9
5. How many 4 digit numbers are there with no digit repeated?
6. How many numbers lying between 100 and 1000 can be formed with the digits 0,1,2,3,4,5, if the repetition of the digits in not allowed.
7. Find the number of different signals that can be generated by arranging at least 2 flag in order (one below the other) on a vertical staff, if 5 different flags are available.
8. How many words, with or without meaning, can be made from the letters of the word MONDAY, assuming that no letter is repeated, if

   (i) 4 letters are used at a time?

   (ii) all letters are used at a time?

   (iii) all letters are used but first letter is vowel?
9. In how many ways 3 maths books, 4 history books, 3 chemistry books and 2 biology books can be arranged on a shelf so that all books of the same subjects are together.
10. In how many arrangements of letter of the word INDEPENDENCE:

    (i) do the words start with P?

    (ii) do the words begin with I and end with P?
11. In how many ways of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together.
12. Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in a dictionary, what will be the 50th word?
13. In how many ways can the letters of the word PERMUATIONS be arranged if there are always 4 letters between P and S?
14. Determine n, if $2n_{c_3}: n_{c_2}$=12:1.

15. **Prove that** $\frac{n_{cr}}{n-1_{c_{r-1}}} = \frac{n}{r}$

16. **What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these are**

(i) face cards?

(ii) cards of the same colour?

(iii) four cards of the same suit?

(iv) four cards belongings to four different suits?

(v) Two red cards and two black cards?

17. **A boy has 3 library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow maths part II, unless Maths part I is also borrowed. In how many ways can he choose the 3 books to be borrowed?**

18. **A group consist of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has (i) no girls? (ii) at least 3 girls. (iii) at least one girl and one boy?**

19. **How many chords can be drawn through 21 points on a circle?**

20. **If there are 20 persons in a party, and if each of them shakes hands with another, how many handshakes happen in the party?**

Principle of Mathematical Induction

I. **For all n ∈ N, prove that**

1. $1^2+2^2+3^2+5^2$ ....$+(2n-1)^2$ = $\frac{n(2n-1)+(2n+1)}{3}$

2. $\frac{1}{2.5}+\frac{1}{5.8}\dots\dots\dots.+\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$

3. 1+4+7+........+ (3n-2)= $\frac{n(3n-1)}{2}$

4. 1+ $\frac{1}{1+2}+\frac{1}{1+2+3}\dots\dots..+\frac{1}{1+2+3+\cdots+n}$ = $\frac{2n}{n+1}$

$$\frac{1}{n+(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$