



DELHI PUBLIC SCHOOL
SAIL TOWNSHIP, RANCHI
ANNUAL EXAMINATION (2017-18)

Class:- XI
Time- 3 Hrs.

Subject:- Mathematics
F. M:- 100

General Instructions:-

- (i) All questions are compulsory.
- (ii) This question paper contains 29 questions.
- (iii) Question 1-4 in section A are very short questions of 1 mark each.
- (iv) Question 5-12 in section B are short questions of carrying 2 marks each.
- (v) Questions 13-23 in section C are long answer type question of 4 marks each.
- (vi) Questions 24-29 in section D are long answer type question of 4 marks each.

Section-A

- Q.1 List all the proper subsets of the set $\{-1, 0, 1\}$
- Q.2 Find limit $x \rightarrow \pi$, $\left(\frac{\sin x}{x-\pi}\right)$
- Q.3 Write negation of the statement " All students study mathematics at the elementary level".
- Q.4 Rewrite in the form "p if and only if q"
"If a quadrilateral is equiangular, then it is a rectangle and if a quadrilateral it is a rectangle, then it is equiangular .

Section-B

- Q.5 Show that for any sets X and Y, $X \cap (X \cup Y) = X$
- Q.6 Solve the inequality $-12 < 4 - \frac{3x}{-5} \leq 2$
- Q.7 If $y = \frac{x}{\cos^n x}$ then find $\frac{dy}{dx}$.
- Q.8 Prove that $n C_r + n C_{r-1} = n+1 C_r$.
- Q.9 If A is the AM and G is GM between two numbers then show that $A \geq G$.
- Q.10 Find the equation of the circle passing through (0,0) and making intercepts a and b on the coordinate axes.
- Q.11 Find the coordinates of the foci, vertices, the eccentricity and the length of the latus rectum of the hyperbola $3y^2 - x^2 = 27$.
- Q.12 Find the mean deviation about the mean of the first n natural numbers when n is an even number.

Section-C

- Q.13. Prove that $\cos \frac{2\pi}{15} \times \cos \frac{4\pi}{15} \times \cos \frac{8\pi}{15} \times \cos \frac{16\pi}{15} = \frac{1}{16}$
OR

If $\tan A \tan B = \sqrt{\frac{a-b}{a+b}}$, prove that $(a-b \cos 2A) (a -b \cos 2B) = a^2 - b^2$

Q.14 Solve the following system of inequation graphically on the same scale

$$x+2y \leq 10$$

$$x+2y \geq 1$$

$$x-y \leq 0$$

$$x \geq 0$$

$$y \geq 0$$

Q.15 Find the value of $\sqrt{1+i}$

OR

If α and β are two different complex numbers with $|\beta| = 1$ then find $\left| \frac{\beta-\alpha}{1-\bar{\alpha}\beta} \right|$

Q.16 In any ΔABC , if $C = 60^\circ$ then prove that

$$\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$$

Q.17 The vertices of a triangle are A(5,4,6) , B (1, -1,3) and C (4,3,2). The internal bisector of ΔABC meets BC in D. Find AD.

Q.18 In an examination a question paper consists of 12 questions divided into two parts i.e. part I and part II , containing 5 and 7 questions respectively A student is required to attempt 8 questions in all , selecting at least 3 from each part. In how many ways can he select the questions?

Q.19 If the p^{th} , q^{th} , r^{th} and s^{th} term of an AP are also in GP then prove that p-q, q-r, r-s are in GP.

Q.20 Find the distance of the point (2,5) from the line $3x+y+4=0$ measured parallel to the line $3x-4y+8=0$

OR

Prove that the product of the length of the perpendiculars drawn from the points $(\pm \sqrt{a^2 - b^2}, 0)$ to the line $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$ is b^2 .

Q.21. Show that the area of triangle formed by lines $y = m_1x + c_1$, $y = m_2x + c_2$ and

$$x = 0 \text{ is } \frac{(c_1 - c_2)^2}{2|m_1 - m_2|} \text{ square units.}$$

Q.22 Find the standard deviation of the following data by step deviation method only

Classes	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

Q.23 The mean and standard deviation of 100 observations were calculated as 40 and 5.1 respectively by a student who took by mistake 50 instead of 40 as one observation. What are the correct mean and correct standard deviation.

Section-D

Q.24. (a) Let f be a subset of $Z \times Z$ defined by $f = \{ (ab, a+b) , a, b \in Z \}$ Is f a function from Z into Z. Justify your answer.

(b) Find the domain and the range of the function $f(x) = \sqrt{9 - x^2}$.

Q.25 Prove that

(a) Evaluate $\lim_{x \rightarrow \frac{\pi}{6}} \left\{ \frac{\sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}} \right\}$

(b) Differentiate $y = \frac{\sin x}{x}$ from the first principle only.

OR

(a) If $y = \frac{x \sin x}{\sin x + \cos x}$ then find $\frac{dy}{dx}$.

(b) Evaluate the limit

$\lim_{x \rightarrow y} \frac{\tan x - \tan y}{x - y}$ (do not use L- Hospital's rule)

Q.26 If the sum of m terms of an AP is equal to the sum of either the next n terms or next p terms then prove that $(m+n) \left(\frac{1}{m} - \frac{1}{p} \right) = (m+p) \left(\frac{1}{m} - \frac{1}{n} \right)$

OR

150 workers were engaged to finish a job in a certain number of days. 4 workers dropped out on second day. 4 more workers dropped third day and so on. It took 8 more days to finish the work. Find the number of days in which the work was completed.

Q.27 If a_1, a_2, a_3, a_4 be the coefficient of four consecutive terms in the expansion of $(1+x)^n$ then prove that

$$\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4} = \frac{2a_2}{a_2+a_3}$$

OR

If P denote the sum of odd terms and Q the sum of even terms in expansion of $(a+x)^n$ then prove that $P^2 - Q^2 = (a^2 - x^2)^n$ and $4PQ = (a+x)^{2n} - (a-x)^{2n}$

Q.28 (a) Find the equation of the parabola which has its axis along x-axis and which passes through the points (3,2) and (-2, -1).

(b) A rod AB of length 15 cm rests in between two coordinate axes in such a way that the end A lies on x-axis and the end B lies on y-axis. A point P(x,y) is taken on the rod in such a way that

AP = 6cm . Show that the locus of p is an ellipse whose equation is $\frac{x^2}{81} + \frac{y^2}{36} = 1$

OR

(a) Prove that the equation of the parabola whose focus is (1,1) and tangent at the vertex is $x+y=1$ is $x^2 + y^2 - 2xy - 4x - 4y + 4 = 0$.

(b) An arch is in the shape of a semi ellipse. It is 8 m wide and 2 m high at the centre. Find the height of the arch at a point 1.5 m from one end.

Q.29 A box contains 6 red, 4 white and 5 black balls . A person draws 4 balls from the box at random. Find the probability that among the balls drawn there is at least one ball of each colour.

