



DELHI PUBLIC SCHOOL

SAIL TOWNSHIP RANCHI

HALF YEARLY EXAMINATION (2017-18)

Class:-XI
Time- 3 Hrs.

Subject:- Mathematics
M.M-100

General Instructions:-

- (i) All questions are compulsory.
- (ii) This question paper contains 29 questions.
- (iii) Questions 1-4 in Section A are very short answer type and carry 1 mark each.
- (iv) Questions 5-12 in Section- B are short type carrying 2 marks each.
- (v) Questions 13-23 in Section -C are long answer type I carrying 4 marks each.
- (vi) Questions 24-29 in Section - D are long answer type II carrying 6 marks each.
- (vii) Please write down the serial number of the question before attempting it.

Section -A

[1x4=4]

1. Solve $3x-7 > 5x - 1$
2. Evaluate $(1+i)^6 + (1-i)^2$
3. Find the radian measure of the angle subtended at the centre of circle of radius 100 c.m by an arc of length 22 c.m (use $\pi = \frac{22}{7}$)
4. The Cartesian product $A \times A$ has 9 elements. Two of the elements are found to be $(-1, 0)$ and $(0, 1)$. Find the set A and remaining elements of $A \times A$.

Section -B

[2x8=16]

5. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$ verify that
(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$
6. Two finite sets have m and n elements respectively. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n.
7. A railway train is travelling on a circular track of 1500 m radius at the speed of 60 k.m/hr. Through what angle in degrees does it turn in 10 seconds?
8. Find the value of $\tan \frac{13\pi}{12}$
9. Solve $|3 - 4x| \geq 9$
10. Find the conjugate of $\frac{(3 - 2i)(2 + 3i)}{(1 + 2i)(2 - i)}$
11. Solve the equation
 $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

12. Evaluate $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x}$

Section -C

[4x11=44]

13. Let A and B be any two sets. If for some set X, $A \cap X = B \cap X = \emptyset$ and $A \cup X = B \cup X$ then prove that $A=B$.

OR

If A and B are any two sets, prove that $P(A) = P(B) \Rightarrow A = B$

14. Let $f = \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in R \right\}$ be a function from R into R . Determine the range of f.

15. Draw the graph of the following functions

(a) $f : R \longrightarrow R$ such that $f(x) = |x - 2|$

(b) $f : R \longrightarrow R$ such that $f(x) = x|x|$

16. Prove that $\tan 6^\circ \times \tan 42^\circ \times \tan 66^\circ \times \tan 78^\circ = 1$

17. If $\sin x = \frac{1}{4}$ and x is in quadrant II

find $\sin \frac{x}{2}$, $\cos \frac{x}{2}$, $\tan \frac{x}{2}$

OR

If $2 \tan \beta + \cot \beta = \tan \alpha$, prove that $\cot \beta = 2 \tan (\alpha - \beta)$

18. Solve the following inequality and show the graph of the solution on number line

$$\frac{x}{2} \geq \frac{5x-2}{3} - \frac{7x-3}{5}$$

OR

Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

19. Prove that $(1 + x)^n \geq (1+nx)$ for all natural numbers n , where $x > -1$

20. Represent the complex number $(-1 + i\sqrt{3})$ in the polar form.

OR

Evaluate $\sqrt{-5 + 12i}$

21. Find the derivative of $\sqrt{\sin x}$ by the first principle

22. If $y = \frac{x \tan x}{\sec x + \tan x}$ then find $\frac{dy}{dx}$.

23. A survey shows that 63 % of the Americans like cheese, where as 76 % like apples. If x% of Americans like both cheese and apples. Find the value of x.

Section -D

[6x6=36]

24. A school awarded 58 medals for honesty, 20 for punctuality and 25 for obedience. If these medals were bagged by a total of 78 students and only 5 students got medals for all the three values. Find the no. of students who received medals for exactly two of three values. Should these awards be included in the regular school curriculum? Comment.

25. If α and β are the solutions of the equation $a \tan x + b \sec x = c$, then show that

$$\tan(\alpha + \beta) = \frac{2ac}{a^2 - b^2}$$

OR

Prove that $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8} = \frac{3}{2}$

26. If z_1 and z_2 both satisfy $z + \bar{z} = 2|z - 1|$ and argument $(z_1 - z_2) = \frac{\pi}{4}$. Find $\text{Im}(z_1 + z_2)$

27. Prove the following by using the principle of mathematical induction for all $n \in N$.

$$1.3 + 3.5 + 5.7 + \dots \dots \dots (2n - 1)(2n + 1) = \frac{n(4n^2 + 6n - 1)}{3}$$

OR

Prove that $2 \cdot 7^n + 3 \cdot 5^n - 5$ is divisible by 24 for all $n \in N$

28. In any ΔABC prove that $a^3 \cos(B-C) + b^3 \cos(C-A) + c^3 \cos(A-B) = 3abc$

29. Solve the following system of inequalities graphically

$$x + 2y \leq 10, \quad x + y \geq 1, \quad x - y \leq 0, \quad x \geq 0, \quad y \geq 0$$

