DELHI PUBLIC SCHOOL

SAIL TOWNSHIP, RANCHI

PRE- BOARD-II EXAMINATION (2017-18)

Class:-XII | **Subject:- Mathematics**

Time- 3 Hrs. | **M.M-100**

General Instructions:-

1. *All questions are compulsory.*
2. *This question paper contains 29 questions.*
3. *Question 1-4 in Section –A are very short answer type questions carrying 1 mark each.*
4. *Question 5-12 in Section –B are short answer type questions carrying 2 marks each.*
5. *Question 13-23 in Section C are long answer –I type questions carrying 4 marks each.*
6. *Question 24-29 in Section –D are long answer – II type questions carrying 6 marks each.*

Section – A **[1x4=4]**

1. Find the number of binary operations on a set A = {a,b}

2. If the matrix A = $[a_{ij}]_{2x2}$, where a_{ij} = 1 if i ≠ j and a_{ij} = 0 if i = j , then compute A^2 .

3. Find the number of vectors of unit length perpendicular to the vectors
$\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$

4. Find the identity element for the binary operation * defined on Q – {0} as
a*b = $\frac{ab}{2}$, ∀ a, b ∈ Q - {0} , where Q- {0} denotes the set of all non – zero rational numbers.

Section – B **[2x8=16]**

5. Solve the equation:- $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

6. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

7. Prove that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x$, if $-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

8. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1 %

9. Evaluate:- $\int \frac{e^{6\log x} - e^{5\log x}}{e^{4\log x} - e^{3\log x}}\, dx$

10. Verify that $ax^2 + by^2 = 1$ is a solution of the differential equation $x\,(yy_2 + y_1^2) = yy_1$
where $y_1 = \frac{dy}{dx}$ and $y_2 = \frac{d^2y}{dx^2}$

11. Find the projection (vector) of $\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$ on the vector $7\hat{\imath} - \hat{\jmath} + 8\hat{k}$

12. If P(A) = 0.8 , P (B) = 0.5 and P (B/A) = 0.4 , find (i) P(A∩B) (ii) P (A/B).

Section – C [4x11=44]

13. Prove that $\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4\, a^2b^2c^2$

14. Evaluate 'k' if

$$f(x) = \begin{cases} \dfrac{2^{x+2}-16}{4^x-16}, & \text{if } x \neq 2 \\ k, & \text{if } x = 2 \end{cases}$$ is continuous at x = 2

OR

Show that the function f(x) = $|x|$ is not differentiable at x = 0

15. If $y = e^{a\cos^{-1}x}$, $-1 \le x \le 1$, show that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0$

16. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

OR

Find the intervals in which the function $f(x) = (x+1)^3 (x-3)^3$ is strictly increasing or strictly decreasing.

17. Evaluate:- $\int \dfrac{(x^4-x)^{1/4}}{x^5}\, dx.$

18. A telephone company in a town has 500 subscribers on its list and collects fixed charges of Rs. 300 per subscriber per year . The company proposes to increase the annual subscription and it is believed that for every increase of Rs. 1, one subscriber will discontinue the service. Find what increase will bring maximum profit. What value is being exhibited by the company?

19. Find the equation of a curve passing through the point (- 2, 3), given that the slope of the tangent to the curve at any point (x,y) is $\frac{2x}{y^2}$

OR

Find the general solution of the differential equation $x\dfrac{dy}{dx} + 2y = x^2\ (x \neq 0)$

20. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$, hence show that $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

21. Find the shortest distance between the lines
$\vec{r} = (\hat{\imath} + 2\hat{\jmath} + \hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ and
$\vec{r} = (2\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$

22. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

23. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?

Section – D [6x6=36]

24. Consider f: $R_+ \longrightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible. Also calculate $f^{-1}(3)$.

OR

$A = N \cup \{0\} \times N \cup \{0\}$ and * be the binary operation on A defined by
$(a,b) * (c,d) = (a+c, b+d) \ \forall \ (a,b), (c,d) \in A$
Show that * is commutative and associative. Find the identity element for * on A, if any.

25. If $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, then find BA and use this to solve the system of equations $y + 2z = 7$, $x - y = 3$ and $2x + 3y + 4z = 17$

OR

If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ find A^{-1}, hence solve $x + 2y + z = 4$, $-x + y + z = 0$, $x - 3y + z = 2$.

26. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ into three equal parts.

27. Evaluate $\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$

OR

Evaluate $\int_0^1 e^{2-3x} dx$ as the limit of sum .

28. Find the length and the foot of the perpendicular from the point (7, 14, 5) to the plane $2x + 4y - z = 2$. Also find the image of the point P in the plane.

29. A toy manufacturer produces two types of dolls; a basic version doll A and a deluxe version doll B. Each doll of type B takes twice as long to produce as one doll of type A. The company has time to make a maximum of 2000 dolls of type A per day, the supply of plastic is sufficient to produce 1500 dolls per day and each type requires equal amount of it. The deluxe version doll requires a fancy dress of which there are only 600 per day available.
If the company makes a profit of Rs. 3 and Rs. 5 per doll on doll A and B; how many of each should be produced per day in order to maximize profit? Solve it graphically.